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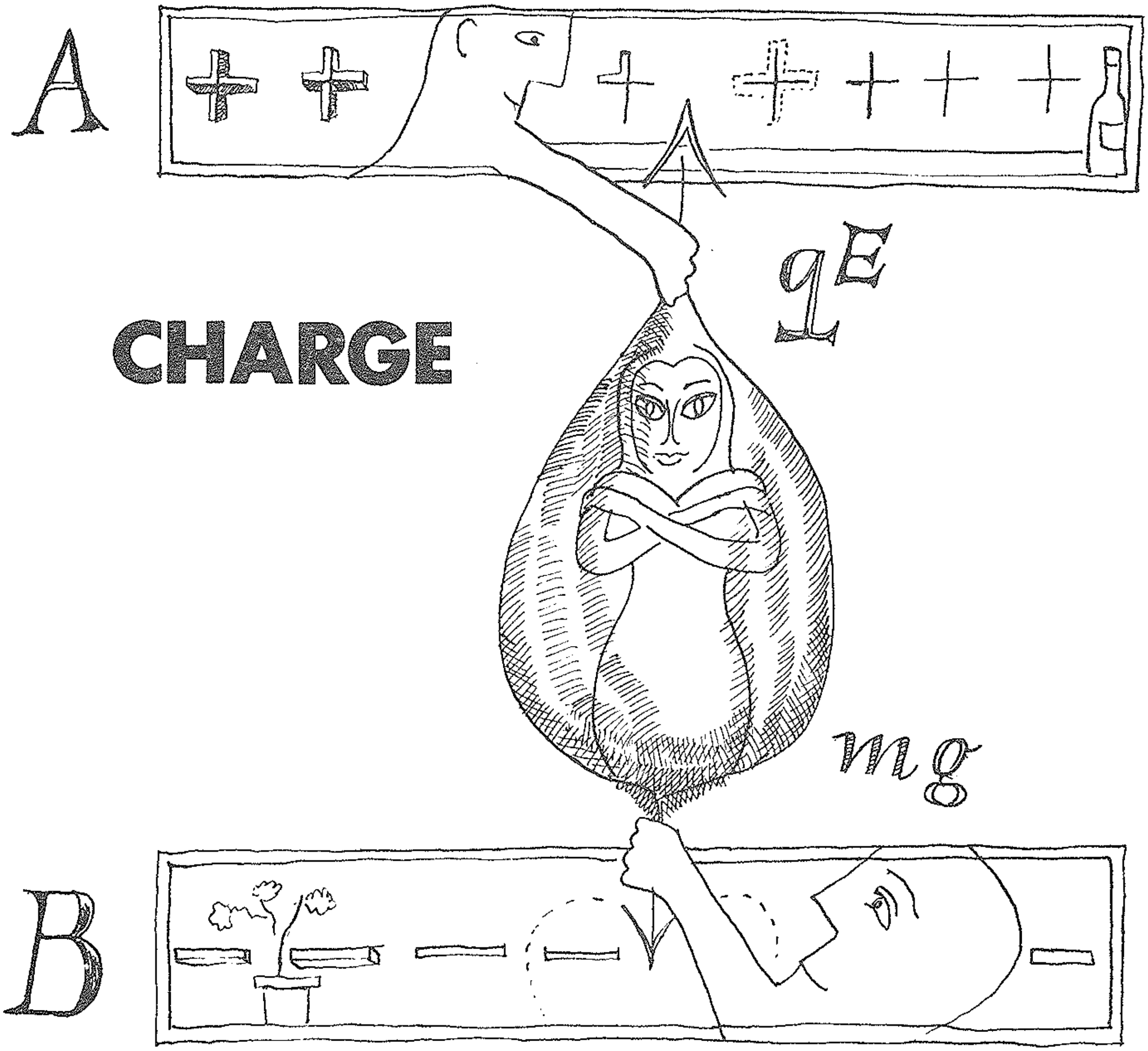

RESOURCE HANDBOOK

HUNTINGTON TWO COMPUTER PROJECT

1 September 1971

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APPENDIX A. Millikan's Oil Drop Experiment: History

The results of J.J. Thomson's electron experiments (1897) and the Rutherford scattering experiments (1909) led to a more general acceptance of the "atomic structure of matter." During the same period many attempts were being made to determine whether electricity had the same properties; that is, did there exist a basic unit of electricity? One of the earliest attempts along this line of inquiry was a series of experiments performed at the Cavendish Laboratory in England by J.J. Thomson.

Just prior to Thomson's investigations, C.T.R. Wilson (also of the Cavendish Laboratory) discovered that air saturated with water vapor will begin condensation on ions introduced into the system. Starting with a saturated mixture,Wilson would ionize air particles by beaming x-rays into the solution. Shortly thereafter, he would expand the volume to create a supersaturated solution which would then condense on the ions created. The complete unit including the volume-expansion mechanism is known as the Wilson "cloud chamber."

Using a cloud chamber, Thomson noted the movements of the droplets formed as they drifted downward under the influence of gravity. He was able to determine the total quantity of water involved in the condensation process by measuring the amounts of water needed to saturate the volume before and after expansion. In addition, Thomson was also able to determine the average size of the droplets by applying a method devised by G.G. Stokes involving the drift of spheres (the droplets) through a viscous medium (the air). Thus, the total number of droplets produced could be determined by dividing the total quantity of water condensed by the mean volume of the droplets formed. By then introducing an electric field in the region, Thomson could separate and collect the droplets with the different charges to determine the total negative or positive charge carried by the drops. These experiments permitted Thomson to determine the average number of charges per drop.

Similar experiments were performed by H.A. Wilson who used a pair of horizontal parallel plates in the cloud chamber to produce a uniform electric field. The field was directed so that it opposed the gravitational force on the negative ions. By varying the voltage applied to the plates, H.A. Wilson was nearly able to balance the drop between the electric and gravitational forces. Determining the average size and mass of the drops as before, he was able to estimate the charge carried by the drops.

In 1911, R.A. Millikan improved Wilson's method by including a precedure for measuring the movements of a single droplet over extended periods of time. Also, noting that the water droplets evaporated rapidly, he designed his experiments to work with oil instead of water. The device used by Millikan is similar to that used in the modern version of the experiment.

Oil from an atomizer was sprayed above the upper plate of the apparatus and some of the oil drops fell through the hole to the region between the plates. The size of the drops could be determined by noting their rate of fall through air, and the mass of each drop determined from the size of the drop and the density of the oil. Once produced, the drops could be ionized by x-rays and prevented fron falling out of view of the observing windows by applying the proper voltage to the metal plates. Actually, when the electric field was applied, some droplets would tend to drift upward while others increased their rate of fall. The speed at which the drops moved depended on the charge they carried and the magnitude of the field. By increasing the voltage applied to the plates, Millikan could sweep the faster moving particles out of view leaving the slower moving particles behind for further observation. Of the drops remaining, Millikan was able to single out one drop and watch it rise and fall under the influence of the various forces.

In the actual experiments, Millikan watched single drops for hours at a time as they rose and fell between two cross-hairs of known separation marked off on his observation telescope. By noting the time required to move across the fixed distance over and over again, Millikan established "means" for the terminal velocity in each direction. Repeating the procedure with many different single oil drops, Millikan obtained values for the charges that fell in well-defined groups; all the charges were integral multiples of a basic quantity. The interpretation of such results was that electric charge is not distributed uniformly, but comes in "clumps" or atoms of electricity" as first postulated by Michael Faraday fifty years earlier.

Once the supporting evidence for a basic electric charge had been established, determination of the number to be assigned to the basic charge could also be found from Millikan's experiment. The terminal velocity under free fall is given in terms of Stokes' constant K and the resultant force acting on the drop. The resultant force F is known to be:

$$F = mg \qquad [1]$$

where m is the mass of the oil drop. Since the density of the oil (d) is known, the force F may also be written as:

$$F = 4/3 \ \pi R^3 dg \qquad [2]$$

With Stokes' constant $K = 6\pi hR$,

$$v_t = F/K = \frac{(4/3)\pi R^3 dg}{6\pi hR} \qquad [3]$$

Solving for R yields:

$$R = \sqrt{\frac{9hv_t}{2gd}} \; . \qquad [4]$$

Once the radius of a given drop is known, Stokes' constant can be calculated and, subsequently, the "basic electric charge."

One of the necessary measurements in Millikan's original experiment was the motion of the oil drop as it fell freely through the air. This measurement permitted the size of the drop and, consequently, the mass of the drop to be calculated. This additional complication can now be eliminated by using tiny latex spheres of known mass and size. (These latex spheres are ordinarily used to calibrate the magnifying power of electron microscopes). The spheres that are commonly available have a diameter of one micron (1×10^{-6} meters) plus or minus about 5%, and a density that is very close to 1000 kg/m^3. The mass of such a sphere is about 5×10^{-16} kilograms. The stability, uniformity and known radius of these spheres make possible a laboratory experiment that is simpler conceptually and easier to do than Millikan's original version. It is this experiment that is simulated in the CHARGE program.

APPENDIX B. The CHARGE Model

I. List of Variables and Constants

A. Variables

		BASIC Notation
1.	v = velocity of the drop in meters/sec	--
2.	V = voltage applied in volts (between -1000 and +1000)	V
3.	c = charge on a drop in coulombs	--
4.	N = number of unbalanced electron charges on a drop	N(I) for drop I

B. Constants

1.	$h = 1.8 \times 10^{-5}$ newton-sec/m^2	viscosity of air at 18°C	H
2.	$g = 9.8$ meter/sec^2	acceleration of gravity	G
3.	$q = 1.6 \times 10^{-19}$ coulomb	electron charge	Q
4.	$d = 1.0 \times 10^3$ kg/m^3	density of latex	D
5.	$R = 5.0 \times 10^{-7}$ meters $\pm$ 1%	radius of drop	R(I)
6.	$D = 2 \times 10^{-2}$ meters	plate separation	D9

II. Equations

A. Drop velocity as a function of voltage

1. $v = F/K$
2. $F = F_e - F_g$ $\qquad F_e = cE$ $\qquad F_g = mg$
3. $K = 6\pi hR$
4. $c = Nq$
5. $E = V/D$
6. $m = 4/3\ \pi R^3 d$

B. Charge calculation at voltage at which $v = 0$

$$cE = mg$$

$$c = mg/E = mgD/V \ .$$

III. Number of Charges per Drop

The number N of charges per drop is a normally distributed random variable with mean and standard deviation both equal to 3. It is determined by using the INT function on the following expression:

$$3 + 3[\cos(2\pi r_1)] \sqrt{-2\ln_e r_2} \qquad (1)$$

where r_1 and r_2 are generated by the RND function.

The function

$$x = [\cos(2\pi r_1)] \sqrt{-2\ln_e r_2} \qquad (2)$$

yields very close to a normal distribution with mean equal to zero and standard deviation equal to 1; with the transformation $y = \sigma x + \mu$, where σ is the standard deviation and μ the mean, we obtain (1). The limits on N in this program are $0 \leq N \leq 8$.

IV. Statistical Variation in Drop Radius

Available latex spheres have a spread in radius from sphere to sphere of about $\pm$ 5%. Since the cube of the radius enters into the equation for the mass of the sphere, the masses can vary by $\pm$ 15%. In the calculation of the charge on a stopped drop, the average mass of a drop is used, since the individual mass is not known. The 15% uncertainty in mass leads to a 15% uncertainty in calculated charge, which masks the discreteness of charge when a sphere contains many charges. (For instance, for a sphere containing 8 charges, the calculated charge can be anywhere between 6.8 (8 - 15%) and 9.2 (8 + 15%). To avoid this problem in the CHARGE program, the radii are assumed to have a variation of only $\pm$ 1%. In the program the radii are uniformly distributed about the average radius R_o by calculating the radius of the Ith drop as:

$$R(I) = R_o(.99 + .02\ \mathrm{RND}(I))$$

where $R_o = 5 \times 10^{-7}$ meters. The differences in radii lead to a spread in drop mass, terminal velocity, and calculated charge.

APPENDIX C. PROGRAM LISTING

```
1ØØ REM N(I)=NUMBER OF ELECTRONIC CHARGE UNITS FOR PARTICLE I
1Ø5 REM COPYRIGHT 1971 - STATE UNIVERSITY OF NEW YORK
115 REM (NORMALLY DISTRIBUTED WITH MEAN 3 AND DEVIATION 3)
12Ø REM M(I)=MASS OF PARTICLE I, BASED ON R(I)
125 REM R(I)=RADIUS OF PARTICLE I
13Ø REM (AVERAGE RADIUS R, 1 PERCENT UNIFORMLY DISTRIBUTED ERROR)
135 REM MODEL DEVELOPED BY A. CAGGIANO AND D. SCARL
14Ø REM PROGRAMMED BY C. LOSIK, JULY 1971
145 REM LATEST REVISION: 8-25-72
15Ø REM Q=RND(-1)
155 RANDOMIZE
16Ø DIM N(4),M(4),R(4)
165 PRINT " ","MILLIKAN OIL DROP EXPERIMENT"
17Ø PRINT
175 REM V:VOLTAGE, INITIALLY ZERO
18Ø LET V=Ø
185 REM H:VISCOSITY COEFFICIENT BETWEEN LATEX SPHERE AND AIR
19Ø LET H=.ØØØØ18
195 REM G:ACCELERATION DUE TO GRAVITY
2ØØ LET G=9.8
2Ø5 REM Q:COULOMBS PER ELECTRONIC CHARGE UNIT
21Ø LET Q=1.6E-19
215 REM D:DENSITY OF LATEX SPHERE
22Ø LET D=1ØØØ
225 REM R:AVERAGE RADIUS OF LATEX SPHERE
23Ø LET R=5.E-Ø7
235 REM D9:PLATE SEPARATION
24Ø LET D9=.Ø2
245 REM P1:VALUE OF 'PI'
25Ø LET P1=3.14159
255 REM M:AVERAGE MASS OF LATEX SPHERE
26Ø LET M=4*P1*R*R*R*D/3
265 REM K1,K2, AND K3 ARE USEFUL CONSTANTS
27Ø LET K1=Q/D9
275 LET K2=G*D9*M
28Ø LET K3=6*P1*H
285 LET NØ=Ø
29Ø PRINT "INSTRUCTIONS (1=YES, Ø=NO)";
295 INPUT I
3ØØ IF I=Ø THEN 36Ø
3Ø5 IF I <> 1 THEN 29Ø
31Ø PRINT
315 PRINT "INSTRUCTIONS -- AFTER EACH QUESTION MARK,";
32Ø PRINT " (V=  ?), YOU MAY:"
325 PRINT
33Ø PRINT "TYPE IN VOLTAGE BETWEEN -1ØØØ AND 1ØØØ (IN ORDER TO MAKE"
335 PRINT "THE VELOCITY PRINTED OUT AS CLOSE TO ZERO AS POSSIBLE),"
34Ø PRINT "REQUEST CALCULATION OF CHARGE FOR STOPPED DROP";
345 PRINT " (TYPE IN 2ØØØ),";
35Ø PRINT "REQUEST NEW BATCH OF DROPS (TYPE IN 3ØØØ),"
355 PRINT "OR END THE PROGRAM (TYPE IN 4ØØØ)."
```

```
360 PRINT
365 PRINT
370 PRINT " ","NO ELECTRIC FIELD"
375 PRINT
380 PRINT "DROP:",N0+1,N0+2,N0+3,N0+4
385 PRINT " ","---","---","---","---"
390 IF N0>0 THEN 415
395 PRINT "  VELOCITY"
400 PRINT "(METERS/SEC)"
405 PRINT "( X  10↑-6 )"
410 REM GENERATE RANDOM CHARGE, RADIUS, AND MASS
415 FOR I=1 TO 4
420 LET N(I)=INT(3+3*COS(6.283*RND(1))*SQR(-2*LOG(RND(1)))+.5)
425 IF ABS(N(I)-4)>4 THEN 420
430 LET R(I)=R*(.99+.02*RND(I))
435 LET M(I)=4*P1*R(I)*R(I)*R(I)*D/3
440 NEXT I
445 PRINT " ",
450 FOR I=1 TO 4
455 PRINT .1*INT(1.E+07*(K1*V*N(I)-M(I)*G)/(K3*R(I))+.5),
460 NEXT I
465 PRINT " "
470 PRINT "V=";V;" ";
475 INPUT I
480 IF ABS(I)>1000 THEN 495
485 LET V=I
490 GOTO 445
495 PRINT
500 IF I <> 2000 THEN 565
505 PRINT
510 PRINT "CALCULATION FOR WHICH DROP";
515 INPUT I
520 FOR J=1 TO 4
525 IF N0+J-I=0 THEN 540
530 NEXT J
535 GOTO 505
540 PRINT
545 PRINT "CHARGE ON DROP";I;" IS";.01*INT(1.E+21*K2/V+.5);
550 PRINT " X 10↑-19 COULOMBS."
555 PRINT
560 GOTO 470
565 IF I <> 3000 THEN 580
570 LET N0=N0+4
575 GOTO 375
580 IF I <> 4000 THEN 470
585 END
```

APPENDIX D. SAMPLE RUNS

```
                 NO ELECTRIC FIELD

DROP:            1              2              3              4
                ---            ---            ---            ---
  VELOCITY
(METERS/SEC)
( X   10↑-6 )
                -30.5          -29.7          -30            -30.7

V= 0    ? 100
                -25.8          -15.4          -20.5          -30.7

V= 100   ? 500
                -7              41.7           17.4          -30.7

V= 500   ? 600
                -2.3            56             26.9          -30.7

V= 600   ? 625
                -1.1            59.5           29.3          -30.7

V= 625   ? 650
                 .1             63.1           31.6          -30.7

V= 650   ? 649
                 0              63             31.5          -30.7

V= 649   ? 2000

CALCULATION FOR WHICH DROP? 1

CHARGE ON DROP 1   IS 1.58   X 10↑-19 COULOMBS.

V= 649   ? 200
                -21.1          -1.1           -11            -30.7

V= 200   ? 210
                -20.6           .3            -10.1          -30.7

V= 210   ? 208
                -20.7           0             -10.2          -30.7

V= 208   ? 2000

CALCULATION FOR WHICH DROP? 2

CHARGE ON DROP 2   IS 4.93   X 10↑-19 COULOMBS.
```

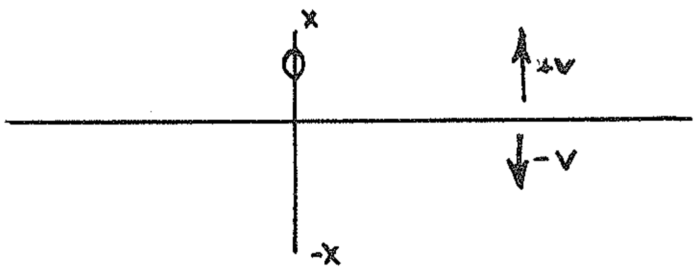
```
V= 208    ? 400
                  -11.7          27.4          7.9          -30.7

V= 400    ? 350
                  -14            20.3          3.2          -30.7

V= 350    ? 325
                  -15.2          16.7          .8           -30.7

V= 325    ? 320
                  -15.4          16            .4           -30.7

V= 320    ? 315
                  -15.7          15.3          -.1          -30.7

V= 315    ? 316
                  -15.6          15.4          0            -30.7

V= 316    ? 2000

CALCULATION FOR WHICH DROP? 3

CHARGE ON DROP 3  IS 3.25  X 10↑-19 COULOMBS.

V= 316    ? 3000

DROP:              5             6             7             8
                  ---           ---           ---           ---
                   74            43.1          -15.5         59.9

V= 316    ? 100
                   2.7           -7.4          -25.6         -1.5

V= 100    ? 90
                  -.6            -9.7          -26.1         -4.4

V= 90   ? 92
                   .1            -9.3          -26           -3.8

V= 92   ? 91
                  -.2            -9.5          -26           -4.1

V= 91   ? 91.7
                   0             -9.3          -26           -3.9

V= 91.7   ? 2000

CALCULATION FOR WHICH DROP? 5

CHARGE ON DROP 5  IS 11.19  X 10↑-19 COULOMBS.
```

```
V= 91.7  ? 150
                    19.2          4.3          -23.3          12.7

V= 150   ? 140
                    15.9          1.9          -23.7          9.9

V= 140   ? 135
                    14.3          .8           -24            8.4

V= 135   ? 132
                    13.3          .1           -24.1          7.6

V= 132   ? 131
                    13            -.2          -24.2          7.3

V= 131   ? 131.7
                    13.2          0            -24.1          7.5

V= 131.7  ? 2000

CALCULATION FOR WHICH DROP? 6

CHARGE ON DROP 6  IS 7.79  X 10↑-19 COULOMBS.

V= 131.7  ? 200
                    35.7          16           -20.9          26.9

V= 200   ? 400
                    101.7         62.7         -11.5          83.8

V= 400   ? 600
                    167.7         109.5        -2.1           140.7

V= 600   ? 625
                    176           115.3        -.9            147.8

V= 625   ? 630
                    177.6         116.5        -.7            149.2

V= 630   ? 650
                    184.2         121.1         .3            154.9

V= 650   ? 640
                    180.9         118.8        -.2            152

V= 640   ? 642
                    181.6         119.3        -.1            152.6

V= 642   ? 643
                    181.9         119.5        -.1            152.9

V= 643   ? 644
                    182.2         119.7         0             153.2

V= 644   ? 2000

CALCULATION FOR WHICH DROP? 7

CHARGE ON DROP 7  IS 1.59  X 10↑-19 COULOMBS.
```

```
V= 644   ? 110
                6             -5.1           -25.2          1.3

V= 110   ? 105
                4.4           -6.2           -25.4          -.1

V= 105   ? 106
                4.7           -6             -25.3          .2

V= 106   ? 15←05.3
                4.5           -6.2           -25.4          0

V= 105.3   ? 2000

CALCULATION FOR WHICH DROP? 8

CHARGE ON DROP 8   IS 9.75   X 10↑-19 COULOMBS.

V= 105.3   ? 4000
```

APPENDIX E. Falling Spheres

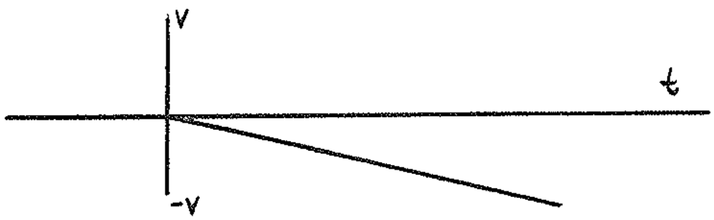
Layer 1:



If a tiny sphere falls in vacuum under the action of gravity alone, the velocity at time t after it starts to fall is

$$v = -gt \ .$$

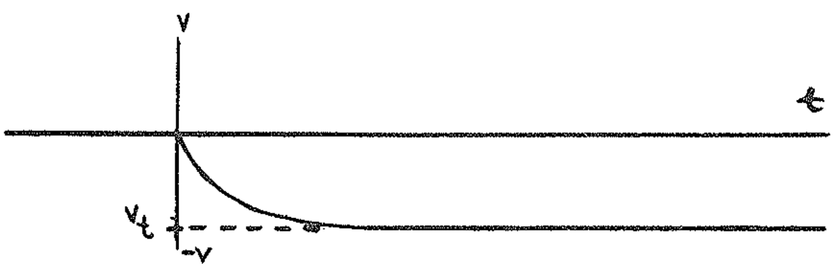
A graph of velocity vs. time looks like:



If the same tiny sphere falls in air under the action of gravity, the velocity at time t is:

$$v = v_t \ .$$

v_t is a constant (the terminal velocity). A graph of velocity vs. time looks like:



The terminal velocity is not reached as soon as the sphere starts to fall, but for the tiny latex spheres in this experiment it takes only about ten microseconds to reach the terminal velocity. The terminal velocity of a tiny sphere can be calculated:

$$v_t = -mg/K$$

where m is the mass of the sphere, g is the acceleration due to gravity, and K has to do with the drag caused by the air. If the radius of the sphere and the viscosity of air are known, K can be calculated. For the spheres used in this experiment and air at room temperature and pressure,

$$K = 1.7 \times 10^{-10} \text{ newton-second/meter}$$

Since the mass of these spheres is 5.2×10^{-16} kg., the terminal velocity is about 2 millimeters per minute.

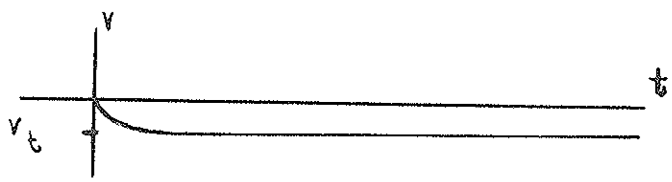
When the electric field is turned on, the terminal velocity of a charged sphere becomes:

$$v_t = (-mg + cE)/K$$

where c is the charge on the sphere and E is the value of the electric field. E can be calculated from the voltage across the plates and the spacing between them by,

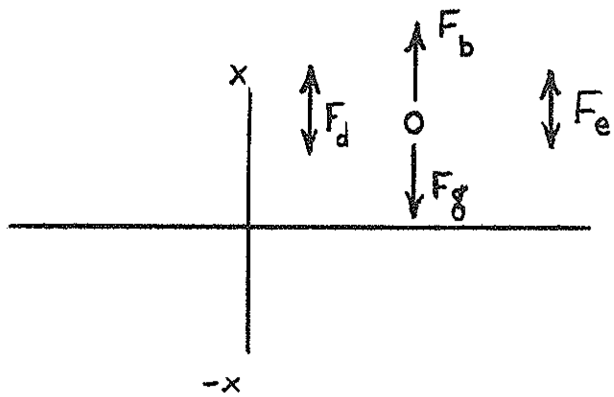
$$E = V/D \ .$$

The graph of velocity vs. time looks like:



When the upward force due to the electric field is just equal to the downward force due to gravity, the velocity is zero and the sphere stops.

Layer 2:



The equation for the motion of the charged sphere is

$$F = ma = m\dot{v}$$

F is the total force on the sphere. It is made up of:

F_g = force of gravity = $-mg$

F_b = bouyant force (since the sphere "floats" in air) = $m_{air}g$,

where m_{air} is the mass of air displaced by the sphere;

F_d = drag force on the sphere due to its motion through the air = $-Kv$,

where K can be calculated from the radius of the sphere and the viscosity of air;

F_e = electric force on the charge due to the electric field = cE.

The bouyant force is less than one percent of the force of gravity and will be ignored here. It must be taken into account in precise determinations of the electronic charge.

The equation can be written:

$$m\dot{v} = F = mg + cE - Kv$$

This differential equation involving both the velocity and the time rate of change of velocity has the solution:

$$v = v_t(1 - e^{-t/t_0})$$

This is the equation that is plotted in the second graph on the first page. The velocity starts at zero and approaches v_t at times long compared with t_o. (When $t = 3t_o$, v is already 95% of v_t.) Since for this form of v

$$\dot{v} = \frac{v_t}{t_o} e^{-t/t_o}$$

v and $\dot{v}$ can be put back into the equation of motion, and the equation will be true if

$$v_t = (-mg + cE)/K$$

and

$$t_o = m/K \ .$$

The calculation of K is a problem in fluid dynamics. In 1843, Stokes found K, the drag force on a sphere falling slowly in a viscous fluid to be:

$$K = 6\pi hR$$

where R is the radius of the sphere and h is the viscosity of air. The general problem of a body moving through a viscous fluid is very complicated and the sphere is one of the few shapes for which K can be calculated at all. Luckily, both oil drops and latex particles are very good spheres.